Group delay tuning in active fiber Bragg gratings: From superluminal to subluminal pulse reflection

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Group delay tuning of optical pulses in pumped asymmetric fiber Bragg gratings below lasing threshold is theoretically proposed. A gain-controlled transition from superluminal to subluminal pulse reflection is analytically investigated in uniform gratings with a π phase shift and in tapered gratings.

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The issue of group delay and group velocity control of electromagnetic pulses, at either microwave or optical frequencies, has received a continuous and increasing interest in the past recent years, and a large body of literature exists on this subject (see, e.g., $[1-3]$ and references therein). A number of schemes have been studied and experimentally demonstrated to achieve group velocity control exploiting coherent light-matter interaction processes, such as electromagnetically induced transparency $[4-7]$ and stimulated Raman processes $[8,9]$, in which the dispersion properties of an atomic medium can be tuned by coupling suitable optical fields between atomic levels. A few recent experiments have nicely demonstrated group velocity control by simple power tuning of laser coupling fields, with a transition from superluminal to subluminal light propagation $[6,7]$. The possibility of controlling light dispersion properties using fiber-based or waveguide-based photonic structures, such as Bragg gratings, coupled-resonator optical waveguides and photonic crystals, has attracted a great attention as well, and experimental demonstrations of slow and fast light effects have been reported by several authors. In these optical structures, however, tuning of the dispersive properties is inherently difficult; a few studies have shown the possibility of tuning pulse group delay exploiting pump-controlled nonlinear effects of the medium, such as phase conjugation in pumped Kerr media [10], quadratic effects in quasi-phase-matched nonlinear crystals [11] and stimulated Brillouin scattering in optical fibers $[12,13]$. Recently, a gain-controlled enhancement of group velocity in transmission has been proposed in a distributed-feedback (DFB) semiconductor amplifier with uniform grating by exploiting the carrier-induced refractive index change of the semiconductor $[14]$. In this case, the basic physical mechanism allowing tuning of the pulse transit time is a gain-induced frequency shift of the Bragg frequency of the grating arising from the semiconductor linewidth enhancement factor related to carrier plasma. As the gain is increased, the gain-induced refractive index change of the semiconductor shifts the carrier wavelength of the optical pulse from the center of the band gap, where the transit time is fast, toward the band gap edge, where the transit time is slowed-down. However, the basic dispersive properties of the grating structure itself *are not structurally changed* by the presence of the gain, in the sense that, e.g., at the band gap center of the grating the group delay remains always

superluminal and at the band gap edges always subluminal: the main role of the gain is to sweep the pulse carrier frequency with respect to the grating spectral curve via a gaininduced refractive index change. In media where gaininduced refractive index changes are negligible, such as in the important class of Bragg gratings written on doped optical fibers or glass waveguides, the group delay tuning mechanism proposed in Ref. $[14]$ cannot be applied, and a consistent change of pulse group delay could be attained at the band gap edges solely for gain levels close to the lasing threshold [15], where the system is very sensitive to noise and to the onset of dynamical instabilities. For such reasons the search for different and more general group delay tuning mechanisms in active Bragg grating structures, operated far below the lasing condition, seems of particular relevance from both physical and applied viewpoints.

In this work a different and rather general mechanism that allows group delay control and tuning of optical pulses in Bragg gratings written on an active optical fiber or glass waveguide, operated below threshold for laser oscillation, is proposed. The main idea is the use of an *asymmetric* grating profile whose dispersive properties *in reflection* can be structurally controlled by varying the gain level of the amplifier yet keeping the amplifier well below lasing threshold. In particular, it is shown that in a wide class of asymmetric structures a gain-controlled transition from superluminal to subluminal pulse reflection can be achieved through a passage from a local zero reflectivity condition, which marks a structural change in the dispersion curve of the grating. The dispersive properties of passive and lossless index-grating Bragg structures with an asymmetric grating profile have been previously studied in $[16]$, and the possibility to achieve superluminal group delays in reflection has been proposed $\lceil 16 \rceil$ and experimentally demonstrated $\lceil 17 \rceil$. On the basis of a Hilbert-like relation between group delay and power reflectivity spectra [Eq. (2) in Ref. [16]], it was shown that in lossless passive gratings the group delay in reflection of a pulse tuned near to a local minimum of the power reflectivity spectrum may be superluminal. In this work we show that, when the medium is active with a uniform gain along the grating, the group delay of the reflected pulse near to a local minimum of the power reflectivity spectrum can be tuned from superluminal to subluminal values by increasing the gain level. The transition from superluminal to sublumi-

FIG. 1. Schematic of pulse reflection in a pumped periodic Bragg grating of length *L*. Pulse reflection is superluminal for τ_r <0 (the case depicted in the figure), and subluminal for τ_r > 2*Ln*₀ / c_0 .

nal regimes occurs through a passage from a local zero reflectivity condition.

The starting point of our analysis is provided by a standard model of Bragg scattering in an active medium with a longitudinal periodic modulation of the refractive index (Fig. 1), which may model, e.g., wave propagation in a Bragg grating written on a pumped erbium-doped fiber or waveguide. For a periodic (i.e., not chirped) grating of length *L*, the refractive index profile has the form $n(z) = n_0$ $+\Delta na(z)\cos(2\pi z/\Lambda_B)$ for $0 < z < L$, where n_0 is the bulk refractive index, Λ_B is the grating period, Δn is the maximum refractive index change, and $a(z)$ is the normalized profile of the grating modulation. If we consider the propagation of a monochromatic field $E(z, t)$ at the optical frequency ω close to the Bragg frequency $\omega_B = c_0 \pi / (n_0 \Lambda_B)$, where c_0 is the speed of light in vacuum, we may write $E(z, t)$ $= u(z, \delta) \exp(-i\omega t + ik_B z) + v(z, \delta) \exp(-i\omega t - ik_B z) + c.c.,$ where $k_B = \pi / \Lambda_B$ is the Bragg wave number and *u*,*v* are the envelopes of counterpropagating waves that, for a shallow grating $(\Delta n \ll n_0)$, satisfy the following coupled-mode equations (see, e.g., [18,19]):

$$
du/dz = (i\delta + g)u + iq(z)v,
$$
 (1)

$$
dv/dz = -(i\delta + g)v - iq(z)u.
$$
 (2)

In Eqs. (1) and (2), $q(z) = [k_B \Delta n/(2n_0)]a(z)$ is the realvalued scattering potential, $\delta = k_0 - k_B = n_0(\omega - \omega_B)/c_0$ is the wave number detuning from the Bragg wave number k_B , and $g > 0$ is the small-signal gain coefficient of the medium, which is assumed to be uniform along the grating and controllable by an external pump field (such as in a pumped Er-doped fiber grating). Since we will consider propagation of low-energy probing pulses with duration much shorter than the relaxation time of the gain medium, gain dynamics and saturation effects of the amplifying medium are not accounted for in our analysis $[20]$. The general solution to Eqs. (1) and (2) can be written as $, v(L)$ ^{*T*} $= M(\tilde{\delta})(u(0), v(0))^T$, where M is the grating transfer matrix whose elements depend on the complex detuning parameter $\delta = \delta - ig$. For a forward-propagating incident pulse (Fig. 1), the appropriate boundary condition is $v(L, \delta) = 0$ and the spectral reflection and transmission coefficients are given by

FIG. 2. (a) Power reflectivity, and (b) group delay in a uniform grating with a π phase shift for $L/L_1=1.7$, $q_0L=2$ and for a few values of the gain coefficient. Curve 1: *gL*= 0; curve 2: $gL= 0.1837$; curve 3: $gL= 0.3551$; curve 4: $gL= 0.4776$. In (b) the group delay is normalized to the grating transit time Ln_0/c_0 . In (c) the behavior of power reflectivity (dashed curve) and normalized group delay (solid curve) versus gain parameter gL is shown at $\delta=0$.

 $r(\tilde{\delta}) = [v(0, \delta) / u(0, \delta)]_{v(L, \delta) = 0} = -\mathcal{M}_{21} / \mathcal{M}_{22}$ and $t(\tilde{\delta})$ $\tilde{\delta}$ $=[u(L, \delta) / u(0, \delta)]_{v(L, \delta)=0} = 1 / \mathcal{M}_{22}$, respectively. The power transmission and reflection spectral coefficients of the grating are then $T(\tilde{\delta}) = |t(\tilde{\delta})|^2$ and $R(\tilde{\delta}) = |r(\tilde{\delta})|^2$. Note that the spectral transmission and reflection coefficients of the active (pumped) grating are simply obtained from those of the passive lossless grating (i.e., for $g=0$) by replacing the real wave number detuning δ with the complex one $\tilde{\delta} = \delta - ig$. From inverse scattering theory, it is known that $r(\tilde{\delta})$, $f(\tilde{\delta})$ $= t(\tilde{\delta}) \exp(-i\tilde{\delta}L)$, and $1/f(\tilde{\delta})$ are causal functions, i.e., they are analytic functions of $\tilde{\delta}$ in the upper half plane Im($\tilde{\delta}$) $> 0, f(\delta) \rightarrow 1$ as $\delta \rightarrow \infty$ and $R(\delta) \le 1$ on the real axis, i.e., for a passive lossless grating. If $\tilde{\delta}_0$ is the pole in the complex plane of $t(\delta)$ with the lowest imaginary part (in modulus), self oscillation from noise of the pumped grating occurs at a gain level $g_{th} = -\text{Im}(\tilde{\delta}_0)$, for which $t(\tilde{\delta}_0) \rightarrow \infty$. In this work we will consider gain levels *below* threshold for self-oscillation, i.e., the active grating is used as an amplifier with distributed feedback and with an injected pulse. In addition, we are interested to study the dependence of time delay of the *reflected* pulse on the gain level, showing the possibility to tune the delay from superluminal to subluminal values. For a spectrally narrow pulse at carrier frequency ω , the delay of

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the reflected pulse may be expressed by the group delay (or phase time) $\tau_r(\omega) = \partial \phi_r / \partial \omega$, where ϕ_r is the phase of the spectral coefficient r [16]. Note that the group delay τ_r accounts for time delay $(\tau_r > 0)$ or time advancement $(\tau_r < 0)$ suffered by the incident pulse after being reflected at the input plane $z=0$ of the grating. Superluminal pulse reflection occurs whenever τ_r <0 [16]; in this case, the peak of the reflected pulse appears *earlier* the peak of the incident pulse has arrived at the input plane, i.e., before it has entered into the grating (see Fig. 1). Conversely, subluminal pulse reflection occurs whenever $\tau_r > 2Ln_0/c_0$, i.e., when the time delay suffered by the pulse becomes larger than the time spent by the pulse to travel forth and back the grating length *L* at the phase velocity c_0 / n_0 . For $g = 0$, it was shown in Ref. [16] that superluminal pulse reflection $(\tau_r < 0)$ may occur in grating structures with an asymmetric profile at frequencies close to a local (nonvanishing) minimum of the spectral power reflectivity *R*, i.e., at a frequency $\omega = \omega_0$ such that $R(\omega_0) > 0$, $(\partial R/\partial \omega)_{\omega_0} = 0$ and $(\partial^2 R/\partial \omega^2)_{\omega_0} > 0$. When the grating is pumped, its dispersive properties can be conveniently tuned. In particular, it may happen that, as the gain *g* is increased, the power reflectivity $R(\omega_0)$ first decreases, vanishes at *g*

 $= g_0$ (with $g_0 < g_{th}$), and then increases. Correspondingly, at the transition $g = g_0$ the group delay $\tau_r(\omega_0)$ changes from superluminal (for $g < g_0$) to subluminal (for $g > g_0$) values, diverging close to the zero reflectivity point, i.e., as $g \rightarrow g_0$ [see, e.g., Fig. $2(c)$ commented on below]. One can prove this scenario for different asymmetric grating profiles. Here we consider specifically two asymmetric profiles $a(z)$ which allow one to analytically calculate in a closed form the spectral reflection coefficient $r(\delta)$: namely a uniform grating with a π phase slip and a tapered grating with an exponential-like profile.

The uniform grating with a π *phase shift*. This structure closely resembles the one used in DFB lasers [18], except that the π phase shift is placed at a position $z=L_1$ close to—but not exactly coincident with—the middle of the grating [16]. We thus have a stepwise behavior for $a(z)$, $a(z)$ $= 1$ for $0 < z < L_1$ and $a(z) = -1$ for $L_1 < z < L$. The transfer matrix $\mathcal{M}(\tilde{\delta})$ of the structure, and hence the reflection and transmission spectral coefficients, can be analytically calculated by standard techniques. One finds

$$
r(\tilde{\delta}) = \frac{i(q_0/\lambda)\sinh(\lambda L_2)[\cosh(\lambda L_1) + i(\tilde{\delta}/\lambda)\sinh(\lambda L_1)] - i(q_0/\lambda)\sinh(\lambda L_1)[\cosh(\lambda L_2) - i(\tilde{\delta}/\lambda)\sinh(\lambda L_2)]}{(q_0/\lambda)^2 \sinh(\lambda L_1)\sinh(\lambda L_2) - [\cosh(\lambda L_2) - i(\tilde{\delta}/\lambda)\sinh(\lambda L_2)][\cosh(\lambda L_1) - i(\tilde{\delta}/\lambda)\sinh(\lambda L_1)]},
$$
\n(3)

where $q_0 = k_B \Delta n / (2n_0)$ and $\lambda = (q_0^2 - \tilde{\delta}^2)^{1/2}$. At zero gain $(g= 0)$, the power reflectivity shows a local minimum at $\delta = 0$, and the group delay τ_r is superluminal (negative) whenever $L_1 > L_2$. A typical behavior of the power reflectivity *R* and normalized group delay $\tau_r c_0 / (Ln_0)$ versus the normalized frequency detuning $\delta L = n_0 L(\omega - \omega_B)/c_0$, for a few values of the gain parameter gL , is shown in Figs. $2(a)$ and 2(b). The threshold for self-oscillation, as obtained by calculation of the poles for Eq. (3), is attained at $g_{th}L \sim 0.753$. The behavior of the power reflectivity and group delay at the Bragg frequency $(\delta = 0)$ versus the gain parameter gL is shown in Fig. $2(c)$. Note that the group delay switches from subluminal to superluminal values at $g_0L \approx 0.265$, where the power reflectivity vanishes [22].

The tapered grating. A second noteworthy example of an asymmetric structure in which the group delay can be tuned from superluminal to subluminal values is that of a tapered grating profile. In order to provide analytical results, we consider as an example the following special profile for the scattering potential:

$$
q(z) = \frac{2\theta a}{\theta \cosh(2\theta z) + \gamma \sinh(2\theta z)} \quad (0 < z < L), \quad (4)
$$

where *a* and γ are arbitrary real-valued positive parameters, with $\gamma > a$, and $\theta = (\gamma^2 - a^2)^{1/2}$. A typical behavior of $q(z)$, shown in Fig. $3(a)$, is that of an exponential-like decaying

profile. This potential, for an infinitely-long grating $(L \rightarrow \infty)$, corresponds to a single-pole (complex Lorentzian) spectral reflection coefficient, and analytical solutions of the scattering problem can be obtained by the application of the Gel'fand-Levitan-Marchenko inverse scattering method [21]. Once the solutions u and v of the coupled-mode equations for the infinitely-long grating are known, application of the boundary condition $v(L, \delta) = 0$ allows one to calculate analytically the transfer matrix of the grating with finite length *L*, and hence its spectral reflection coefficient, which reads explicitly

$$
r(\tilde{\delta}) = \frac{a(\tilde{\delta} - i\gamma + i\alpha)\exp(-i\tilde{\delta}L) - \beta(\tilde{\delta} - i\gamma)\exp(i\tilde{\delta}L)}{(\tilde{\delta} + i\gamma)(\tilde{\delta} - i\gamma + i\alpha)\exp(-i\tilde{\delta}L) - \beta a\exp(i\tilde{\delta}L)},
$$
\n(5)

where we have set

$$
\alpha = \frac{a^2 \sinh(2\theta L)}{\theta \cosh(2\theta L) + \gamma \sinh(2\theta L)},
$$
\n(6)

$$
\beta = \frac{\theta a}{\theta \cosh(2\theta L) + \gamma \sinh(2\theta L)}.
$$
 (7)

A typical behavior of power reflectivity and group delay for this kind of grating is shown in Fig. 3. Figure $3(b)$ shows the behavior of power transmission $|t|^2$ in the complex $\tilde{\delta}$ plane;

FIG. 3. (a) Profile of a tapered grating [Eq. (4)] for $\gamma L = 1$ and $aL = 0.99$. (b) Behavior of grating transmission $|t|^2$ in the $(\delta L, gL)$ complex plane. (c) Power reflectivity R (dashed curve) and normalized group delay $\tau_r c_0 / (L n_0)$ (solid curve) versus δL for the unpumped tapered grating (gL=0). (d) Behavior of power reflectivity (dashed curve) and normalized group delay (solid curve) versus gain parameter *gL* for $\delta L = \pm 3.23$.

the existence of the two poles depicted in the figure indicates that the threshold for self-oscillation for this grating is reached at $g_{th}L \approx 1.69$. For the passive lossless grating $(g=0)$, the power reflectivity profile *R* versus normalized wave number detuning δL shows several local and nonvanshing minima at frequencies symmetrically detuned from the Bragg frequencies $\delta = 0$, and the group delay at these frequencies is superluminal [see Fig. $3(c)$]. As the gain *g* is increased, at these frequencies the behavior of the power reflectivity and group delay is analogous to that previously found for the uniform grating with a phase defect, i.e., a transition from superluminal to subluminal pulse delays occurs at the gain level g_0 where the power reflectivity vanishes. This is shown in Fig. $3(d)$ for the case of the two frequencies closest to the Bragg frequency, i.e., for $\delta L \approx \pm 3.23$; the transition occurs in this case at $g_0 L \approx 0.53$.

In order to provide realistic parameters for an experimen-

FIG. 4. Normalized intensity profiles of reflected pulses in a $L=1$ -cm-long π phase shifted grating for $gL=1.59$ dB (dashed curve), corresponding to superluminal reflection, and $gL = 3.08$ dB (solid curve), corresponding to subluminal reflection. The dotted curve shows the intensity profile of the incident Gaussian pulse. Peak power reflectivity is 3.83% and 9.8% for the superluminal and subluminal pulses, respectively.

tal observation of pulse delay tuning from superluminal to subluminal regimes, let us consider as an example reflection from a unform grating with a π phase shift written on an Er-doped fiber or waveguide (refractive index $n_0 \approx 1.46$) with a Bragg wavelength (in vacuum) λ_B = 1550 nm. We assume a grating length $L=1$ cm, so that the characteristic transit time of the grating is $Ln_0/c_0 \approx 48.7$ ps. For L_1 =5.88 mm and for a refractive index change Δn ≈ 1 × 10⁻⁴, this grating corresponds to that shown in Fig. 2. Assuming an incident unchirped Gaussian pulse with a pulse duration (FWHM in intensity) $\tau = 1$ ns, the intensity profiles of the reflected pulses for gain levels $g=0.1837$ cm⁻¹ \approx 1.59 dB/cm and *g*=0.355 cm⁻¹ \approx 3.08 dB/cm, corresponding to curves 2 and 3 in Fig. $2(b)$, are shown in Fig. 4. We note that such gain levels can be achieved in a 1-cm-long grating using recently developed erbium-doped phosphate fibers $[23]$ or glass waveguides $[24]$. As a final remark, it should also be noted that, since the pumped grating is operated at rather low amplifying levels [see Fig. $2(a)$] and in the unsaturated regime [20], amplification of the noise level should be negligible.

In conclusion, we have theoretically shown that pulse delay tuning from superluminal to subluminal values can be achieved in active Bragg gratings with an asymmetric profile by exploiting a structural change of the dispersive reflective curve at a frequency close to a local minimum of power reflectivity. An experimental demonstration of group delay tuning based on such a different tuning mechanism could be easily achieved using Er-doped pumped fiber gratings.

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